The triple pomeron interaction in the perturbative QCD

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Abstract.

The triple pomeron interaction is studied in the perturbative approach of BFKL-Bartels. At finite momentum transfers $\sqrt{-t}$ the contribution factorizes in the standard manner with a triple-pomeron vertex proportional to $1/\sqrt{-t}$. At $t = 0$ the contribution is finite, although it grows faster with energy than for finite t and does not factorize.

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1 Introduction

There has been much theoretical activity around the triple pomeron interaction, [1 - 4] related to recent experimental data on the diffractive virtual photon scattering on HERA [5]. In fact, the corresponding diffractive differential cross-section involves many different contributions, of which the triple pomeron is only the simplest one. All sorts of corrections can be added in the framework of the standard Regge-Gribov theory, which include other pomerons or/and m to n pomeron verteces. With the contribution of a single pomeron exchange rising with energy, none of these unitarity corrections can be safely considered as small. The situation improves in the perturbative approach. In the perturbative QCD one can study all contributions to the diffractive cross-section ("effective triple pomeron vertex") in the leading order in the small (fixed) coupling constant [2]. The cross-section turns out to be a rather complicated mixture of the triple pomeron interaction proper and the transitions of four interacting gluons into two pomerons, which makes its practical calculation hardly possible.

Further improvement can, however, be achieved if along with a small coupling constant one assumes a large number of colours $N \to \infty$. Then, as one easily finds, the triple-pomeron interaction indeed becomes the dominant contribution, all others damped either by a factor $1/N^2$ (transitions 4 gluons $\to$ 2 pomerons) or by a supposedly small factor g^2N (m to n pomeron verteces other than the triple one). One may hope that these assumptions are not too far from reality. The triple pomeron interaction thus retains some practical interest, to say nothing of its theoretical importance, as one of the basic quantities in the perturbative QCD at high energies and small x.

Since in the BFKL theory [6] the pomeron is nonlocal and consists of two interacting reggeized gluons, the triple pomeron vertex is essentially a 2 to 4 gluon vertex in this theory. This vertex $K_{2\to 4}$ was found by J.Bartels quite a time ago [7] (and rederived for $t = 0$ by a different method much later in [8]). To obtain the triple pomeron vertex from $K_{2\to 4}$ all one has to do is to integrate it with three BFKL pomerons coupled to external sources and find the asymptotical behaviour of the resulting expression at high energies where the pomerons exist. However this straightforward (although requiring some care) program has not, as far as we know, been carried out up to now, for reasons beyond our comprehension. The only calculation of this sort in [8] refers only to $t = 0$ and is not explicit. The present note is devoted to fill this gap.. We calculate the triple pomeron interaction in the BFKL-Bartels approach, both at zero and nonzero momentum transfer $\sqrt{-t}$. The result is finite in both cases although the point $t = 0$ is singular. For $t < 0$ the contribution factorizes in the standard manner into three pomerons coupled by the triple pomeron vertex $\gamma_{3P}(t) = const/\sqrt{-t}$.

The numerical constant which determines the magnitude of γ_{3P} is given by a complicated 6-dimensional momentum integral (Eq. (40)), which we unfortunately have not been able to calculate up to now. At $t = 0$ the triple pomeron contribution results much more complicated and does not factorize into pomerons and their interaction vertex. It also rises with energy faster than at $t < 0$.

Comparing our results to the literature, we find much similarity to the triple pomeron vertex at $t < 0$ of A.Mueller an B.Patel [4], obtained on the basis of the original equation for single and double colour dipole densities proposed by A.Mueller in [9]. In particular the $1/\sqrt{-t}$ behaviour is the same, which, however, does not mean much, since this behaviour trivially follows from dimensional arguments. It is the numerical factor that matters. This factor is different in [4]. As we shall argue (see Section 5.) it corresponds to a different physical picture, which, in our opinion, does not agree with the s-channel unitarity.

2 The triple pomeron interaction in the BFKL-Bartels formalism

Consider the 3→3 amplitude A shown in Fig. 1. Let $s = (p_1 - p_2)^2$, $s_1 \equiv M^2 = (p_1 + p_2 - p_3)^2$, $l = p_3 - p_2$, $t = l^2$. We shall study the s_1 discontinuity of A in the triple Regge region $s, s_1 \to \infty$, $s >> s_1$, t finite. It can be expressed via the unitarity relation in the s_1 channel

$$\mathrm{Disc}_{s_1} A \equiv 2iD = i \sum_n \int d\tau_n |A(3, n|1, 2)|^2 \tag{1}$$

The amplitude $A(3, n|1, 2)$ refers to the process in which n additional gluons are produced with momenta $k_1, ...k_n$ (Fig. 2); τ_n is the corresponding phase space volume. In contrast to the analogous amplitude which appears in the construction of the BFKL pomeron [6], in our case the lowest t-channel is colourless. Our normalization is that the inclusive diffractive cross-section to produce particle 3 with a missing mass M^2 is given by

$$\frac{d\sigma}{dt dM^2} = \frac{D}{16\pi^2 s^2} \tag{2}$$

Such an amplitude with $n = 1$ has been extensively studied by J.Bartels in [7] who has shown that in the leading order in the coupling constant g it can be found from its $s_2 = (p_3 + k)^2$ discontinuity. Suitably generalizing his result to n produced gluons we assume that in the multiregge kinematical region the amplitude $A(3, n|1, 2)$ can be represented as shown in Fig. 3. Its upper part corresponds to a reggeized gluon which emits intermediate real gluons. The lower part represents a pair of interacting reggeized gluons in a colourless state, which form a pomeron. The transitional vertex V which describes emission of the gluon k, the upper reggeon splitting into two lower ones, has been found in [7]. It consists

of two terms, a local and a nonlocal ones, both expressed through the standard Lipatov reggeon-reggeon-particle vertex [6]. We shall not need its explicit form here.

Putting Fig. 3 into the unitarity relation we find that the upper reggeized gluon and its conjugated partner also form a pomeron, so that the discontinuity D can be graphically represented by Fig. 4. It shows three pomerons coupled to colourless external sources and joined in the center by a transitional vertex $K_{2\rightarrow 4}$. The latter is obtained by squaring the Bartels vertex V and summing over polarizations of the intermediate gluon k. Explicitly, $K_{2\rightarrow 4}$, apart from a colour factor and a factor g^4, is given by

$$K_{2\rightarrow 4}(q_1,-q_1;q_2,l-q_2,-l+q_3,-q_3)\equiv K_l(q_1,q_2,q_3)=$$
$$\frac{q_1^2q_2^2}{(q_1-q_2)^2}+\frac{q_1^2q_3^2}{(q_1-q_3)^2}-\frac{q_1^4(q_2-q_3)^2}{(q_1-q_2)^2(q_1-q_3)^2} \tag{3}$$

Translating Fig. 4 into an expression for D we find

$$D=(1/8)g^{10}N^2(N^2-1)(s^2/s_1)\int\prod_{i=1}^{3}(d^2q_i/(2\pi)^3)K_l(q_1,q_2,q_3)$$
$$\phi_1(s_1,0,q_1)\phi_2(s_2,l,q_2)\phi_2(s_2,-l,-q_3);\quad s_2=s/s_1 \tag{4}$$

Here $\phi_{1(2)}(s,l,q)$ is a pomeron (a solution to the BFKL equation) coupled to the projectile (target) colourless external source with energy squared s, total momentum l and one of the gluon's momentum q. It is assumed that the colour factor for each source is $(1/2)\delta_{ab}$ and that each source is proportional to g^2. The factor $g^{10}/8$ combines the g-dependence from the sources and the vertex K and also factors 1/2 for each source from its colour structure.

Note that the BFKL equation determines $\phi(s,l,q)$ up to terms proportional to $\delta^2(q)$ or $\delta^2(l-q)$. This circumstance is of no importance when the pomeron is coupled to colourless sources at both ends, since they vanish at $q=0$ or $q=l$. However, in our case this is only true for gluons with momenta q_2 and $-q_3$: the vertex $K_l(q_1,q_2,q_3)$ is equal to zero when $q_i=0,\ i=1,2,3$ but it does not vanish if $q_2=l$ or $q_3=l$. The physical solution, as obtained from iterations of the BFKL equation, does not possess any singularities of this type. So in the following we shall have to adopt special measures to eliminate the δ type singularities at $q_2=l$ or $q_3=l$.

Since for $l\neq 0$ the solution of the BFKL equation is easier to obtain in the (transversal) coordinate space, we pass to this space by presenting

$$\phi(s,l,q)=\int d^2r\phi(s,l,r)\exp(ir(q-l/2)) \tag{5}$$

Evidently r is the transvercal distance between the gluons. Then (4) transforms into

$$D=(1/8)g^{10}N^2(N^2-1)(s^2/s_1)(2\pi)^{-3}\int\prod_{i=1}^{3}d^2r_iK_l(r_1,r_2,r_3)$$

$$\exp(-il(r_2+r_3)/2)\phi_1(s_1,0,r_1)\phi_2(s_2,l,r_2)\phi_2(s_2,-l,-r_3) \tag{6}$$

where the 2 to 4 reggeon vertex in the coordinate space is obtained from (3) to be

$$K_l(r_1,r_2,r_3) = -2(\delta^2(r_2)\delta^3(r_3)\nabla_1^2\delta^2(r_1)+$$

$$\delta^2(r_3)r_2^{-2}(r_2\nabla_1)\delta^2(r_1)+\delta^2(r_2)r_3^{-2}(r_3\nabla_1)\delta^2(r_1)+r_2^{-2}r_3^{-2}(r_2r_3)\nabla_1^4\delta^2(r_1)) \tag{7}$$

The solutions ϕ vanish when $r = 0$. So only the last term in (7) survives. We then can rewrite (6) as

$$D = (1/4)g^{10}N^2(N^2-1)(s^2/s_1)\int(d^2q/(2\pi)^3)\chi_1(s_1,0,q+l/2)(\nabla_q\chi_2(s_2,l,q))^2 \tag{8}$$

where

$$\chi_1(s,0,q) = \int d^2r\nabla^4\phi_1(s,o,r)\exp iqr \tag{9}$$

and

$$\chi_2(s,l,q) = \int d^2rr^{-2}\phi_2(s,l,r)\exp iqr \tag{10}$$

The solutions $\phi_{1(2)}$ can be obtained by using the Green function of the BFKL equation for a given total momentum $G_l(s,r,r')$. For the projectile (see [10]):

$$\phi_1(s,l,r) = \int(d^2r'/(2\pi)^2)(G_l(s,r,0)-G_l(s,l,r')\ \rho_1(r') \tag{11}$$

Here $\rho_1(r)$ is the colour density of the projectile as a function of the intergluon distance with the colour factor $(1/2)\delta_{ab}$ and g^2 separated. For the projectile we choose a real or virtual transverse photon with $p_1^2 = -Q^2 \leq 0$, which splits into $q\bar{q}$ pairs of different flavours. The explicit form of ρ is well-known for this case [10]:

$$\rho_1(r) = (2/\pi)\alpha_{em}\sum_{f=1}^{N_f} Z_f^2\int_0^1 d\alpha(m_f^2\mathrm{K}_0^2(\epsilon_f r)+(\alpha^2+(1-\alpha)^2)\epsilon_f^2\mathrm{K}_1^2(\epsilon_f r)) \tag{12}$$

where $\epsilon_f^2 = Q^2\alpha(1-\alpha)+m_f^2$ and m_f and Z_f are the mass and charge of the quark of flavour f. For $Q^2 >> m_f^2$ one can neglect the quark masses and evidently

$$\rho_1(r) = Q^2\tilde{f}_1(Qr) \tag{13}$$

where $\tilde{f}_1(r)$ is a dimensionless function which behaves as $1/r^2$ at the origin and exponentially falls at large r. For a real photon ($Q = 0$) ρ_1 has the same structure, with $q \to m$, where m is of the order of the lightest quark mass

$$\rho_1(r) = m^2 f_1(mr) \tag{14}$$

The dimensionless f_1 has the same properties as $\tilde{f}_1$. As for the target, we shall not specify the explicit form of its colour density $\rho_2(r)$. We shall only assume that the corresponding

mass scale is of the order m and much less than Q for a virtual projectile, so that the scaling property (14) is valid for it.

Eqs (8)-(11) completely determine the triple pomeron interaction. In what follows we calculate the asymptotical behaviour of D at large s_1 and s_2. To finally fix our normalizations we present here the $2 \to 2$ amplitude in the single pomeron exchange approximation in our notations

$$A(s,t) = (i/4)g^4(N^2-1)s\int (d^2rd^2r'/(2\pi)^4)\rho_1(r)\rho_2(r')G_l(s,r,r') \tag{15}$$

The elastic cross-section is given by

$$d\sigma^{el}/dt = (1/16\pi s^2)|A(s,t)|^2 \tag{16}$$

3 The triple pomeron vertex at $t < 0$

We begin with the calculation of the function $\chi_1(s,0,q)$ defined by (9) and (11). Due to the singular character of $\nabla^4\phi$ as a function of r and the subtraction at $r' = 0$ in (11) we shall need two leading terms in the asymptotical expansion of χ_1 at large s, so that we have to actuate with some precision. The BFKL Green function at $l = 0$ and large s is given by the expression [11]

$$G_0(s,r,r') = (1/8)rr'\int_{-\infty}^{\infty}\frac{d\nu s^{\omega(\nu)}}{(\nu^2+1/4)^2}(r/r')^{-2i\nu} \tag{17}$$

where

$$\omega(\nu) = (g^2N/2\pi^2)(\psi(1) - \mathrm{Re}\psi(1/2+i\nu)) \tag{18}$$

Small values of ν play the dominant role in (17) at large s, so that we can approximate

$$\omega(\nu) = \omega_0 - a\nu^2;\quad \omega_0 = (g^2N/\pi^2)\ln 2,\quad a = (7g^2N/2\pi^2)\zeta(3) \tag{19}$$

(ω_0 is the pomeron intercept minus one). Applying to (17) the operator ∇^4 we obtain

$$\nabla^4G_0(s,r,r') = (2s^{\omega_0}r'/r^3)\int_{-\infty}^{\infty}d\nu s^{-a\nu^2}(r/r')^{-2i\nu} \tag{20}$$

To integrate over r in (9) we shift the integration contour in ν from the real axis to a parallel line C with $\mathrm{Im}\nu > 1/2$. Then (20) becomes integrable over r:

$$I_1 \equiv \int d^2r\exp(iqr)\nabla^4G_0(s,r,r') = \pi s^{\omega_0}qr'\int_C d\nu s^{-a\nu^2}(qr'/2)^{2i\nu}\Gamma(-1/2-i\nu)/\Gamma(3/2+i\nu) \tag{21}$$

Now we return to the integration over the real axis. In doing so we come across a pole at $\nu = i/2$, which gives a separate contribution:

$$I_1 = 4\pi^2s^{\omega_0+a/4} + \pi s^{\omega_0}qr'\int_{-\infty}^{\infty}d\nu s^{-a\nu^2}(qr'/2)^{2i\nu}\Gamma(-1/2-i\nu)/\Gamma(3/2+i\nu) \tag{22}$$

The first term, coming from the pole, gives the leading contribution at $s \to \infty$. It does not however depend on r' and is eliminated by the subtraction in (11). The second term is standardly evaluated by the stationary point method at large s:

$$-4\pi s^{\omega_0}\sqrt{\frac{\pi}{a\ln s}}\exp(-\frac{\ln^2 cqr'}{a\ln s}) \tag{23}$$

The number c is determined from the equation for the stationary point to be

$$c = 2\exp(-2-\psi(1)) \tag{24}$$

Putting (23) into (11) we obtain

$$\chi_1(s,0,q) = 4\pi q s^{\omega_0}\sqrt{\frac{\pi}{a\ln s}}\int(d^2r/(2\pi)^2)r\rho_1(r)\exp(-\frac{\ln^2 cqr}{a\ln s}) \tag{25}$$

Passing to the integration over r in (25), let us first assume that the projectile photon has a large virtuality Q^2. Using (13) we change the integration over r to that over the dimensionless Qr

$$\chi_1(s,0,q) = (4\pi q s^{\omega_0}/Q)\sqrt{\frac{\pi}{a\ln s}}\int(d^2r/(2\pi)^2)r\tilde{f}_1(r)\exp(-\frac{\ln^2 cqr/Q}{a\ln s}) \tag{26}$$

The integral over r is now well convergent both at small and large r. So r takes on finite values. We shal later see that for $l \neq 0$ the integhral over q is also convergent and average values of q are of order l. Therfore we may drop all factors in the logarithm squared in the exponent, except $1/Q$. This finally gives

$$\chi_1(s,0,q) = 4\pi q s^{\omega_0}(\tilde{b}_1/Q)\sqrt{\frac{\pi}{a\ln s}}\exp(-\frac{\ln^2 Q}{a\ln s}) \tag{27}$$

where the number $\tilde{b}_1$ is defined by

$$\tilde{b}_1 = \int(d^2r/(2\pi)^2)r\tilde{f}_1(r) \tag{28}$$

Note that $R_1 = \tilde{b}_1/Q$ has the meaning of the transverse dimension of the projectile. From (12) we find

$$\tilde{b}_1 = (3\pi\alpha_{em}/256)\sum_f Z_f^2$$

The same argument evidently applies to the case when the projectile photon is real and $Q = 0$. Then one uses (14) and obtains a similar formula with $Q \to m$ and $\tilde{f}_1 \to f_1$. Since m is finite one can also drop the exponential factor, so that for a real photon

$$\chi_1(s,0,q) = 4\pi q s^{\omega_0}(b_1/m)\sqrt{\frac{\pi}{a\ln s}} \tag{29}$$

where b_1 is defined as in (28) with $\tilde{f}_1 \to f_1$. For a real photon

$$b_1 = (3\alpha_{em}/64)m\sum_f Z_f^2/m_f$$

Now we turn to the function $\chi_2(s,l,q)$. The leading contribution to the BFKL Green function at $l\neq 0$ has the form [11]

$$G_l(s,r,r')=(1/4\pi^2)\int\frac{d\nu\nu^2}{(\nu^2+1/4)^2}s^{\omega(\nu)}E_l^{\nu}(r)E_l^{-\nu}(r') \tag{30}$$

where

$$E_l^{\nu}(r)=\int d^2R\exp(ilR)(\frac{r}{|R+r/2||R-r/2|})^{1+2i\nu} \tag{31}$$

It is evident that the Green function (30), transformed into momentum space, contains terms proportional to $\delta^2(l/2\pm q)$, which should be absent in the physical solution (this circumstance was first noted by A.H.Mueller and W.-K.Tang [12]). For that, (30) goes to zero at $r=0$. Terms proportional to $\delta^2(l/2+q)$ are not dangerous to us: they are killed by the vertex $K_{2\to 4}$, as noted in the previous section. To remove the dangerous singularity at $q=l/2$ and simultaneously preserve good behaviour at $r=0$ we therefore make a subtraction in E, changing it to

$$\tilde{E}_l^{\nu}(r)=\int d^2R\exp(ilR)((\frac{r}{|R+r/2||R-r/2|})^{1+2i\nu}-|R+r/2|^{-1-2i\nu}+|R-r/2|^{-1-2i\nu}) \tag{32}$$

This subtraction removes the δ singularity at $q=l/2$ and doubles the one at $q=-l/2$, the latter eliminated by the kernel $K_{2\to 4}$

Integration over r leads to the integral

$$J(l,q)=\int(d^2r/(2\pi)^2)(1/r^2)\tilde{E}_l^{\nu}(r) \tag{33}$$

This integral is convergent at any values of ν, the point $\nu=0$ included, when the convergence at large values of r and R is provided by the exponential factors. So in the limit $s\to\infty$, when small values of ν dominate, we can put $\nu=0$ in J:

$$J(l,q)=\int(d^2Rd^2r/(2\pi)^2)\frac{\exp(ilR+iqr)}{r|R+r/2||R-r/2|}+(1/l)\ln\frac{|l/2-q|}{|l/2+q|} \tag{34}$$

The second term comes from the subtraction terms in (32). Passing to the Fourier transform of the function $1/r$ one can represent (34) as an integral in the momentum space

$$J(l,q)=(1/l)\int\frac{d^2p(l+|l/2+p|-|l/2-p|)}{(2\pi)|l/2+p||l/2-p||q+p|} \tag{35}$$

In the same manner we can put $\nu=0$ in the function $E_l^{-\nu}(r')$ obtaining for the integral over r'

$$\int\frac{d^2Rd^2r\exp(ilR)r\rho_2(r)}{(2\pi)^2|R+r/2||R-r/2|}=(\pi/m)F(t) \tag{36}$$

Here we have separated the characteristic dimensional factor $1/m$ (and a factor π for convenience) and introduced a dimensionless function $F_2(t)$ which is a vertex for the interaction of the target with a pomeron at momentum transfer $\sqrt{-t}$. $F_2(t)$ behaves like $\ln t$ at small t.

The rest of the Green function is easily calculated by the stationary point method to finally give

$$\chi_2(s,l,q) = 8s^{\omega_0}(\pi/a\ln s)^{3/2}(1/m)F_2(t)J(l,q) \tag{37}$$

Combining our results for χ_1 and χ_2, from (8) we obtain our final expression for the triple-pomeron interaction at $t<0$

$$D = 8\pi^{3/2}g^{10}N^2(N^2-1)(s^2/s_1)\frac{(s_1 s_2^2)^{\omega_0}}{sqrta\ln s_1(a\ln s_2)^3}$$

$$(\tilde{b}_1/Q)\exp(-\frac{\ln^2 Q}{a\ln s_1}(F_2(t)/m)^2 B/\sqrt{-t} \tag{38}$$

Here the number B is defined as a result of the q integration

$$B = l\int(d^2q/(2\pi)^2)|l/2+q|(\nabla_q J(l,q))^2 \tag{39}$$

It does not depend on l and can be represented as an integral over three momenta

$$B = (1/(2\pi)^4 l)\int d^2q d^2p d^2p' \frac{|l/2+q|(q+p)(q+p')(l+p_+-p_-)(l+p'_+-p'_-)}{p_+p_-p'_+p'_-|q+p|^3|q+p'|^3} \tag{40}$$

where

$$p_\pm = |p\pm l/2|;\quad p'_\pm = |p'\pm l/2| \tag{41}$$

It is a well-defined integral, although not quite easy to calculate.

To interprete various factors entering (38) we compare it with the expressions for the elastic amplitude (15) both at $t=0$ and $t<0$ which follow from similar calculations. At $t=0$, for a virtual photon projectile,

$$A(s,0) = i(1/2)g^4(N^2-1)s^{1+\omega_0}(\tilde{b}_1 b_2/mQ)\sqrt{\frac{\pi}{a\ln s}}\exp(-\frac{\ln^2 Q}{a\ln s}) \tag{42}$$

At $t<0$, for a real photon projectile

$$A(s,t) = i(1/2)g^4N^2s^{1+\omega_0}\frac{\sqrt{\pi}}{(a\ln s)^{3/2}}(F_1(t)F_2(t)/m^2) \tag{43}$$

Evidently one cannot determine the pomeron and its couplings in a unique way from these expressions. We may take that the pomeron contribution is

$$P(s,t) = 2\sqrt{\pi}s^{1+\omega_0}(a\ln s)^{-\epsilon(t)} \tag{44}$$

with $\epsilon(0)=1/2$ and $\epsilon(t)=3/2$ for $t<0$. Then taking $N>>1$ we find for its coupling γ to a real external particle at $t=0$

$$\gamma(0) = (1/2)g^2Nb/m \tag{45}$$

and at $t<0$

$$\gamma(t) = (1/2)g^2NF(t)/m \tag{46}$$

For a coupling $\tilde{\gamma}$ to a virtual photon at $t = 0$ we find

$$\tilde{\gamma} = (1/2)g^2 N(\tilde{b}/Q)\exp(-\frac{\ln^2 Q}{a\ln s}) \tag{47}$$

(it is not local in rapidity).

Rewriting (38) in terms of these quatities we obtain, say, for a virtual projectile

$$D = \tilde{\gamma}_1\gamma_2^2(t)P(s_1,0)P^2(s_2,t)\gamma_{3P}(t) \tag{48}$$

with the triple-pomeron vertex $\gamma_{3P}(t)$ given by (for $N >> 1$)

$$\gamma_{3P}(t) = 8g^4 NB/\sqrt{-t} \tag{49}$$

Thus at $t < 0$ the triple pomeron interaction factorizes in the standard manner into three pomerons coupled to external sources and joined by a triple pomeron vertex, independent of energies and proportional to $1/\sqrt{-t}$, which dependence follows trivially from dimensional considerations.

The triple pomeron interaction looks singular at $t = 0$: both the vertex γ_{3P} and couplings γ to external particles diverge at $t = 0$. However we shall presently see that the interaction is, in fact, finite at $t = 0$, although it grows with energy faster than (48).

4 The triple pomeron interaction at $t = 0$

At $t = 0$ we have to calculate the function χ_2 in a different manner, since the integrals (33) and (36) cease to converge. We take the Green function (17) and integrate it over r, as indicated in (10)

$$I_2 \equiv \int d^2r\exp(iqr)r^{-2}G_0(s,r,r') = (\pi r'/4q)s^{\omega_0}\int\frac{d\nu s^{-a\nu^2}(qr')^{2i\nu}}{(\nu^2+1/4)^2}\frac{\Gamma(1/2-i\nu)}{\Gamma(1/2+i\nu)} \tag{50}$$

With $l = 0$, $\nabla_q\chi_2 = (q/|q|)(\partial/\partial q)\chi_2$. So we actually need the derivative of (50) with respect to q. Calculating the asymptotics at large s we find

$$(\partial/\partial q)I_2 = -(4\pi r'/q^2)s^{\omega_0}\sqrt{\frac{\pi}{a\ln s}}\exp(-\frac{\ln^2 c_1qr'}{a\ln s}) \tag{51}$$

We do not need to shift the integration contour in ν here, since the integral is convergent at small r. The number c_1, determined from the stationary point equation, is

$$c_1 = 2\exp(-1-\psi(1)) \tag{52}$$

Integrating (51) over r' with the external source we find

$$(\partial/\partial q)\chi_2(s,0,q) = -(4\pi/q^2)s^{\omega_0}\sqrt{\frac{\pi}{a\ln s}}\int(d^2r/(2\pi)^3)r\rho_2(r)\exp(-\frac{\ln^2 c_1qr}{a\ln s}) \tag{53}$$

Putting (25) and (53) into (8) we obtain the triple pomeron contribution in the form of an integral over the projectile and target transverse dimensions

$$D(t=0) = 2g^{10}N^2(N^2-1)(s^2/s_1)s_1^{\omega_0}s_2^{2\omega_0}\sqrt{\frac{\pi}{a\ln s_1}}\frac{\pi}{a\ln s_2}$$

$$\int\prod_{i=1}^{3}(d^2r_ir_i/(2\pi)^2)\rho_1(r_1)\rho_2(r_2)\rho_2(r_3)W(r_1,r_2,r_3) \tag{54}$$

where W denotes the integral ober q:

$$W(r_1,r_2,r_3) = \int(d^2q/(2\pi)^2)q^{-3}\exp(-\frac{\ln^2 cqr_1}{a\ln s_1} - \frac{\ln^2 c_1qr_2}{a\ln s_2} - \frac{\ln^2 c_1qr_3}{a\ln s_2}) \tag{55}$$

It is easily calculated to give

$$W = (1/2\pi)\sqrt{\pi a_0}(r_1/e)^{\alpha_1}(r_2r_3)^{\alpha_2}\exp(a_0/4)$$

$$\exp(\frac{a_0}{a_1a_2}(2\eta_1(\eta_2+\eta_3-\eta_1)-\eta_2^2-\eta_3^2)-\frac{a_0}{a_2^2}\ln^2(r_2/r_3)) \tag{56}$$

where we have denoted, for brevity,

$$\eta_1 = \ln cr_1,\ \ \eta_2 = \ln c_1r_2,\ \ \eta_3 = \ln c_1r_3$$

$$a_1 = a\ln s_1,\ \ a_2 = a\ln s_2,\ \ a_0 = a\ln s_1\ln s_2/\ln s_1s_2^2$$

$$\alpha_1 = \ln s_2/\ln s_1s_2^2,\ \ \alpha_2 = \ln s_1/\ln s_1s_2^2,\ \ \alpha_1+2\alpha_2 = 1$$

From (56) we observe that at $t=0$ the triple pomeron interaction, although finite, grows faster with energy than at $t<0$ due to the first exponential factor. It leads to an additional power growth, whose strength depends on the relation between s_1 and s_2. It is maximal when $s_1\sim s_2\sim\sqrt{s}$ and this factor is $\sim s^{a/24}$. One also notes that the dimensional factor is composed from r_1, r_2 and r_3 in a proportion which also depends on the relation between s_1 and s_2.

In the integration over r's we, as before, assume that the only large scale Q may be involved in the virtual projectile. Then in the second exponential in (56) we can drop all terms except η_1 which may be substituted by $-\ln Q$. This leads to our final expression

$$D(t=0) = (\pi/a)g^{10}N^2(N^2-1)(s^2/s_1)s_1^{\omega_0}s_2^{2\omega_0}\sqrt{\frac{1}{\ln s_2\ln s_1s_2^2}}(eQ)^{-1-\alpha_1}m^{-2(1+\alpha_2)}$$

$$\tilde{b}_1(\alpha_1)b_2(\alpha_2)\exp(\frac{a\ln s_1\ln s_2}{4\ln s_1s_2^2} - \frac{2\ln^2 Q}{a\ln s_1s_2^2}) \tag{57}$$

where dimensionless $\tilde{b}_1(\alpha)$ and $b_2(\alpha)$ are defined in analogy with (28) with an extra power of r:

$$\tilde{b}_1(\alpha) = \int(d^2r/(2\pi)^3)r^{1+\alpha}\tilde{f}_1(r) \tag{58}$$

and similarly for $b_2(\alpha)$.

The expression (57) has a different structure as compared with (47) at $t<0$. It does not factorize into three pomerons and their interaction vertex independent of energies. In fact the dependence of (57) on s_1 and s_2 is quite complicated and also enters the target and projectile factors b. However this loss of factorization should only occur at very small values of $\sqrt{-t}$. In the integral (55) very small values of q dominate: $\ln(1/q)\sim\sqrt{\ln s}$,i.e. $q\sim m\exp(-\sqrt{a\ln s})$. Recalling that at $t<0$ characteristic values of q are of order l, we come to the conclusion that the transition from (47) to (57) and the breakdown of factorization occurs at

$$\sqrt{-t}<m\exp(-\sqrt{a\ln s}) \tag{59}$$

5 Discussion

We have calculated the triple pomeron interaction in the perturbative QCD picture, based on the gluon reggeization and s-channel unitarity (the BFKL-Bartels approach). The important result is that, contrary to some pessimistic judgements [13], the triple pomeron vertex is infrared finite both at $t=0$ and $t<0$. The subtraction (32) in the pomeron Green function has been essential in this respect.

It is instructive to compare our result (at $t<0$) to that of A.H.Mueller and B.Patel [4], which was obtained in a different framework, based on original evolution equations for single and double dipole densities [9]. We shall not attempt to discuss these equations in general here, limiting ourselves with only the triple pomeron, as obtained in [4]. At $t<0$ the triple pomeron interaction of A.H.Mueller and B.Patel is the same as our Eq. (38) (for a real projectile) with a different numerical factor:

$$B\rightarrow(1/2\pi^2)V_0 \tag{60}$$

where, in our notation,

$$V_0=(l/\pi)\int(d^2r_2d^2r_3/(2\pi)^2)\exp(-il(r_2+r_3)/2)\frac{E_l^0(r_2)E_l^0(r_3)}{|r_2+r_3|r_2^2r_3^2} \tag{61}$$

Turning to our derivation, it is easy to check that this result is obtained if, instead of the Bartels vertex (7), one uses

$$\tilde{K}_l(r_1,r_2,r_3)=-(\pi/4)\frac{\delta^2(r_1+r_2+r_3)}{r_1^2r_2^2r_3^2}+$$
$$\textit{terms proportional to }\delta^2(r_i),\ i=1,2,3 \tag{62}$$

This simple vertex has the form which one might naively write from dimensional arguments, assuming also symmetry in the interacting pomerons. It is, however, singular in the ultraviolet

and does not admit transition to the momentum space. It is evidently different from the Bartels vertex (7) obtained from the s-channel unitarity. Thus it looks as if the triple pomeron of A.H.Mueller and B.Patel would not correspond to the s-channel unitarity.

6 Acknowledgements

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8 Figure captions

Fig. 1. The $3 \to 3$ amplitude whose discontinuity in $M^2 = (p_1 + p_2 - p_3)^2$ contains the triple pomeron interaction.

Fig. 2. The production amplitude $A(3, n|12)$ which enters the unitarity equation (1).

Fig. 3. The production amplitude $A(3, n|1, 2)$ in the multiregge kinematical region. The upper part shows a reggeized gluon emitting real gluons. The lower part shows two interacting reggeized gluons, which form a pomeron. The two parts are joined by a transitional vertex V.

Fig. 4. The triple pomeron interaction as a result of a transition from two reggeized gluons to four. The central line, with the emission of two additional gluons from a point, corresponds to the Bartels vertex $K_{2\to 4}$, Eq. (3).

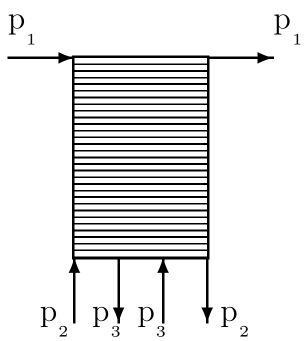

Fig. 1

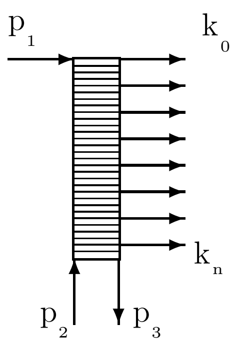

Fig. 2

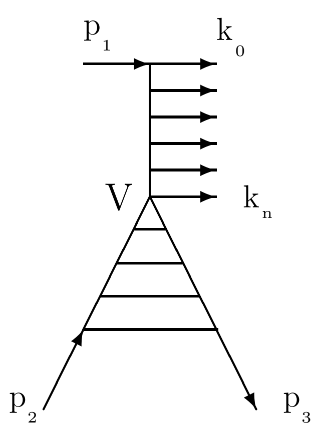

Fig. 3.

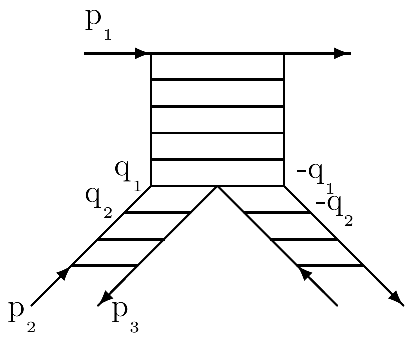

Fig. 4